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A supermultiplier model with two non-capacity-generating semi-autonomous demand components

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ABSTRACT

According to supermultiplier models, economic dynamics are driven by the dynamics of non-capacity-generating autonomous demand components. Since the literature suggests several candidates for these components (government expenditures, credit-financed consumption, private residential investments, etc.), it is necessary to analyze how two (or more) autonomous components can coexist, which is the purpose of the theoretical model, conceptual discussion and simulation exercises presented herein. Because no more than one component can remain exogenous in the long run, either the other component(s) must adjust endogenously, or all the components adjust to each other. In the latter case, the long-run growth rate becomes path dependent, a new finding in the literature on supermultiplier models. Furthermore, while the label 'autonomous' becomes questionable for qualifying demand components, we argue that they should not be considered induced either, leading us to opt for the 'semi-autonomous' label suggested by Fiebiger (2018, 2020) and Fiebiger and Lavoie (2019).

1. Introduction

In recent years, there has been a growing interest in supermultiplier models, in which the dynamics of production and capital accumulation are driven by the dynamics of non-capacity-generating autonomous demand components, while the rate of capacity utilization converges to its normal value.[1] The literature proposes many candidates for the role of autonomous components, such as government expenditures (Allain, 2015; Hein, 2018; Freitas and Christianes, 2020; Hein and Woodgate, 2021; Morlin, 2022), credit-financed consumption (Freitas and Serrano, 2015; Serrano and Freitas, 2017), capitalist consumption (Lavoie, 2016; Nah and Lavoie, 2019a, 2019b), private residential investment (Fiebiger, 2018; Fiebiger and Lavoie, 2019), essential goods (Allain, 2019, 2021), consumption out of wealth (Brochier and Silva, 2019), exports (Nah and Lavoie, 2017; Morlin, 2022), etc. If these numerous candidates illustrate the potential interest of supermultiplier models, then they nevertheless raise the question of the coexistence of several autonomous components in the same model. Indeed, it is obvious that two autonomous components cannot grow indefinitely at different paces, as the component with the lower growth rate will eventually be eliminated. However, is it possible for two (or more) components to be autonomous while having the same growth rate? This question, which has gone largely unnoticed in the literature, is the main purpose of this article, which develops and analyzes the 'long-run' properties of a simple but 'general' supermultiplier model with two non-capacity-generating autonomous demand components. By 'general', we mean that the two components remain unspecified, which will help to keep the focus on the general properties of the model. Moreover, the term 'long run' here refers to the theoretical horizon in which all adjustments following an initial exogenous shock have been made, assuming that the system is not subject to any other disturbance.

Our argument can be summarized as follows. Only endogenous mechanisms can explain the mutual convergence of the growth rates of the different components. Therefore, two types of long-run equilibria can emerge: either the steady growth path is given by the growth rate of a single autonomous component (which could lead to considering the other components as induced), or the growth rates of all components become endogenous, which results in a path-dependent steady growth

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[1] The first formulation of the model was developed by Serrano (1995a, 1995b) and was then completed by Allain (2015) who introduced the mechanism that enables the convergence of the rate of capacity utilization to its normal level.

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path. The article explores both options.

Of course, the long-run steady growth path is unlikely to ever be achieved in a real economy that is continuously affected by new disturbances. Despite its rather restrictive conclusions, however, the long-run analysis allows a wide range of possibilities for applied analysis. Indeed, the theoretical model suggests that economic dynamics in a given period of time can be driven by a single autonomous component, or by several autonomous components which are in conflict to each other; some components can be autonomous in some periods but induced in others; the identity of the driving components (government expenditure, exports, residential investment, etc.) can change from period to period; some periods can be subject to path dependency but not others; etc.

Our analysis is based in part on an attempt to clarify the concepts of 'induced' and 'autonomous' components. Authors who are skeptical of supermultiplier models, such as Skott (2017,2019)7 and Nikiforos (2018), criticize the autonomous nature of demand components in the long run. On the contrary, we try to further explore the claim that "'autonomous' need not mean 'exogenous' or 'constant'" (Fazzari et al., 2020, p. 20), and provide arguments to adopt the suggestion made by Fiebiger (2018, 2020) and Fiebiger and Lavoie (2019) to replace the term 'autonomous' with 'semi-autonomous'.

Our approach also makes it possible to examine and provide some answers to three other criticisms of supermultiplier models. First, Skott (2019) considers the theoretical predication that an increase in the growth rate of an autonomous component results in a decrease in its income share as counterintuitive. Second, Skott (2017) also questions the speed of the convergence mechanism: the adjustment must be slow enough to avoid Harrodian instability, but this lengthens the time before the model reaches its steady-state equilibrium. Third, Blecker and Setterfield's (2019) claim that "supermultiplier analyses have prompted a sudden, late, and undesirable turn towards *exogenous* growth theory in heterodox macrodynamics" (p. 366, italics added).

The rest of the article is organized as follows. Section 2 gives a first glance at the definition of induced and autonomous components. We then present a supermultiplier model including only one autonomous component (Section 3), which allows for further conceptual considerations about induced, autonomous and semi-autonomous components (Section 4). The distinction of several non-capacity-generating autonomous components is introduced in Section 5. Then, Section 6 gives the numerical calibration of the simulation exercises that are presented and discussed in Sections 7 and 8. Section 9 is devoted to concluding remarks.

2. Induced and autonomous components in the short run

By distinguishing between the short, medium, and long run, the modeler usually distinguishes the parameters or demand components that are assumed to be endogenous from those that are assumed to be exogenous. This section focuses only on the short run, while it is extended to the long run in Section 4. The distinction between the 'induced' and 'autonomous' components is commonly accepted as the basis of the principle of effective demand. Thus, according to Cesaratto (2016, p. 45), "the Keynesian logic is that, given productive capacity, the autonomous components of aggregate demand or injections in old-fashioned jargon (typically autonomous consumption, investment, government spending, and exports) determine the level of output, while induced consumption and leakages (saving, taxation, and imports) are a result of the income multiplier process".

We assume here that aggregate demand (Y) is composed of three components

$$Y = C + I + Z \tag{1}$$

where C, I and Z represent induced consumption, gross investment, and a non-capacity-generating autonomous component, respectively. Induced consumption is endogenous since it is assumed to depend on the current income level (Y) as follows:

$$C = (1 - s)Y \tag{2}$$

where s represents the propensity to save, which is assumed here to be exogenous. In Eq. (2), the causality directly runs from current income to consumption because consumption results from the decision of households on the allocation of their income: part of it is consumed through domestic products, and the rest (sY) leaks out in the form of saving and other leakage (taxes and imports). Moreover, as shown in Eq. (2), induced consumption and leakage are usually specified as a proportion of the current level of income, which means that their growth rates are given by the growth rate of income. Therefore, using Serrano's (1995a, p. 70) words, induced consumption is positively engaged in "the circular flow of income" which is the core of the multiplier process.

Conversely, the non-capacity-generating autonomous component (Z) relates to exogenous injections into the multiplier process. According to Serrano (1995a), this would refer to "all those expenditures (whether formally classified as consumption or as 'investment') that are neither financed by the contractual (wage and salary) incomes generated by production decisions" (p. 71). Serrano then offers a long list of components, including capitalist consumption, residential investment of households, government consumption, exports, etc. More recently, Freitas and Serrano (2015) emphasize that autonomous components are "unrelated to the *current* level of output resulting from firms' production decisions" (p. 261, italics added), a definition that is also used by Dejuán (2017, p. 372) and Brochier and Silva (2019, p. 414), among others. Referring to the fact that induced consumption is usually specified as a proportion of the current income level as in Eq. (2), Allain (2021) suggests that "the autonomous components must be defined as all demand components that are not induced, that is, components whose level is not proportional to the current level of the aggregate income" (p. 619).

To preserve the generality of the framework, we do not specify the type of expenditures captured by Z (government consumption, capitalist consumption, residential investment of households, exports, etc.). Eq. (1) indicates that these components are financed by the part of saving and leakage that is not devoted to investment ($Z = Y - C - I$). However, we cannot be more specific about how they are funded. In particular, the part financed by leakage (taxes and import) cannot be distinguished from the part financed by debt.[2]

As noted in the introduction, authors who are skeptical of the supermultiplier framework (Skott, 2017, 2019; Nikiforos, 2018) agree that several demand components are autonomous in the short run, but they question the assumption that they remain autonomous in the long run. One of the main purposes of the article will be to propose a clarification.

Gross investment (I) is assumed to be the only capacity-generating demand component of the model. Its specification constitutes the main (if not the only) difference between the Sraffian and neo-Kaleckian versions of supermultiplier models. Sraffian authors (Serrano, Freitas, Brochier, Morlin, etc.) consider gross investment an induced component that is determined by

$$I = hY \tag{3}$$

where $h \in [0, 1]$ represents both firms' propensity to invest and the investment-to-output ratio. For many post-Keynesians, however, there is no reason why investment should neglect animal spirits and thus be totally induced, at least in the short run. Instead, they prefer the behavioral assumption used in neo-Kaleckian models (see Allain, Fiebiger, Hein, Lavoie, Nah, etc.) according to which gross investment is

[2] On the introduction of financial issues in supermultiplier models, see, among others, Pariboni (2016), Brochier and Silva (2019), Freitas and Christianes (2020), Mandarino et al. (2020), Hein and Woodgate (2021).

partly autonomous and partly induced: on the one hand, investment depends on both the secular (or long-run) rate of capital accumulation expected by entrepreneurs (γ) and the exogenous rate of capital depreciation (δ); on the other hand, it depends on the gap between the actual value of the rate of capacity utilization ($u = Y/Y_{fc}$ is the ratio of actual to full-capacity output) and its normal value (u_n). We thus have

$$I = [\gamma + \delta + \gamma_u(u - u_n)]K \tag{4}$$

where K represents the capital stock and $\gamma_u > 0$ is an exogenous parameter. Referring to Cesaratto's quote above, $(\gamma + \delta)K$ represents the amount of new injections into the system in each period, regardless of the current economic situation. In contrast, the term $\gamma_u(u - u_n)K$ shows how investment is modulated according to the current pressure on capacities: any change in the level of current output leads to a change in the value of u (because $u = \nu Y/K$) that changes the current amount of investment, which, in turn, leads to a change in the level of current output, etc. Therefore, the induced part of investment is engaged in the circular flow of income at work in the multiplier process in the same way as the induced consumption or leakage. We retain this neo-Kaleckian version in the rest of the article.

3. Model resolution considering one non-capacity-generating autonomous demand component

Substituting Eqs. (2) and (4) into (1) leads to the short-run equilibrium rate of capacity utilization:

$$u = \frac{\nu(\gamma + \delta - \gamma_u u_n + z)}{s - \nu\gamma_u} \tag{5}$$

where $z = Z/K$ and $\nu = K/Y_{fc}$ represents the capital to full-capacity output ratio. The denominator of Eq. (5) is assumed to be positive for the Keynesian stability condition to be fulfilled. Moreover, the numerator is assumed to be positive but lower than the denominator so that $0 < u < 1$.[3]

Unsurprisingly, the comparative static analysis confirms the usual post-Keynesian outcomes: the paradox of thrift occurs because an increase in the propensity to save implies a decrease in both u and the rate of capital accumulation ($g_K = I/K$); similarly, any increase in γ (corresponding to animal spirits) or in the autonomous component (through z) entails an increase in both u and g_K.

According to the principle of effective demand, the model dynamics depend on those of the aggregate demand components. From Eq. (2), it is clear that the dynamics of induced consumption are endogenously given by the growth rate of current income (g_Y) as long as the propensity to save remains constant.

With respect to the dynamics of gross investment, notably, neo-Kaleckian models have been subject to two important criticisms when they are applied to long-run analyses. First, Eq. (4) oddly suggests that firms could be satisfied with a situation in which the values of the actual and normal rates of capacity utilization remain permanently different from one another. Second, this equation suggests that firms do not mind if their expectation of the secular rate of accumulation (γ) differs persistently from its current level (g_K). A joint solution to these criticisms is to assume that entrepreneurs adjust their expectations to close the gap between these two rates:[4]

$$\dot{\gamma}_t = \psi(g_{Kt-1} - \gamma_{t-1}) = \psi\gamma_u(u_{t-1} - u_n) \tag{6}$$

where $\psi < 1$ corresponds to the adjustment speed. The behavior described in Eq. (6) generates Harrodian knife-edge instability. However, as shown first by Allain (2015), this instability can be tamed by the stabilizing properties of supermultiplier effects, provided that the value of the ψ parameter remains sufficiently low, which is a point that is discussed below, when the model is calibrated.

The dynamics of the non-capacity-generating autonomous component (Z) are given by its growth rate, g_Z, which is assumed to be exogenous for now. The critical question of whether or notg_Z can be endogenous in the long run is deferred to the following sections.

The model dynamics consist of a succession of short-run moving equilibria that satisfy Eq. (5) at each period. By differentiating Eq. (5), the dynamics between two consecutive periods are given by

$$\dot{u} = \frac{\nu(\dot{\gamma} + \dot{z})}{s - \nu\gamma_u} \tag{7}$$

where $\dot{z} = z(g_Z - g_K)$. Of course, every deviation in capacity utilization affects the rate of capital accumulation through Eq. (4) and entrepreneurs' expectations through Eq. (6).

Most of the above dynamics involve the transition (the traverse) toward a long-run equilibrium. This is reached when the rate of capacity utilization stabilizes ($\dot{u} = 0$), which requires that $\dot{\gamma} = \dot{z} = 0$. Therefore, the long-run equilibrium of the model is characterized by[5]

$$u^* = u_n \tag{8}$$

$$g_Y^* = \gamma^* = g_K^* = g_Z \tag{9}$$

The growth rate in the steady state is thus given by the growth rate of the non-capacity-generating autonomous component, g_Z. Moreover, investment, which was assumed to be partly autonomous in the short run, becomes induced in the long run. Indeed, as for induced consumption, the pace of capital accumulation is now given by the growth rate of income (g_Y^*).

Furthermore, substituting Eqs. (8) and (9) into Eq. (5) yields another property of the long-run equilibrium, which is

$$z^* = \frac{su_n}{\nu} - (g_Z + \delta) \tag{10}$$

The local stability conditions of the system result from the analysis of the following Jacobian matrix, where the partial derivatives are calculated at the long-run equilibrium:

$$J(\gamma^*, z^*) = \begin{pmatrix} \frac{\psi\gamma_u\nu}{s - \nu\gamma_u} & \frac{\psi\gamma_u\nu}{s - \nu\gamma_u} \\ -\frac{sz^*}{s - \nu\gamma_u} & -\frac{\gamma_u\nu z^*}{s - \nu\gamma_u} \end{pmatrix} \tag{11}$$

The determinant of J, $Det(J) = \psi\gamma_u\nu z^*/(s - \nu\gamma_u)$, is unambiguously positive (provided that the Keynesian stability condition holds). Consequently, the local stability condition depends on the sign of the trace, $Tr(J) = \gamma_u\nu(\psi - z^*)/(s - \nu\gamma_u)$, which is negative provided that

$$\psi < z^* \tag{12}$$

In addition, substituting $K = Z/z^*$ and Eq. (8) into Eq. (6) yields

$$Y^* = \frac{1}{s - (g_Z + \delta)\frac{\nu}{u_n}} Z \tag{13}$$

where the ratio is 'the' supermultiplier. Note that its interpretation is more restrictive than the usual concept of multiplier, which measures the impact on Y of any shift in an autonomous component. Indeed, Eq. (13) applies for a specific value of g_Z, provided that all the long-run

[3] The Sraffian approach leads to a different specification of the short-run equilibrium. Substituting Eqs. (2) and (3) into (1) results in: $u = \nu z/(s - h)$.

[4] A dot over a variable is used to indicate a time variation ($\dot{x}_t = x_t - x_{t-1}$). The dynamics are presented in discrete time as in the below simulations. The second equation here is obtained by substituting Eq. (4) into the first one and assuming a constant rate of capital depreciation.

[5] Asterisks denote the long-run equilibrium values of the corresponding endogenous variables.

adjustments have been completed according to Eqs. (8) and (9). Therefore, the supermultiplier only measures the impact on Y of a change in Z that is exactly given by g_Z. Moreover, Eq. (13) shows that an increase in the propensity to save results in a decrease in the income level. Therefore, although s has no impact on u^* and g_Y^*, an amended version of the paradox of thrift still occurs in the long run.

Eq. (13) also enables us to calculate the long-run equilibrium share of autonomous consumption in output:

$$\left(\frac{Z}{Y}\right)^* = s - \frac{\nu}{u_n}(g_Z + \delta) \tag{14}$$

while the long-run equilibrium investment share in output corresponds to the complement to unity of the sum of the C-to-Y ratio (which is equal to $1 - s$) and the Z-to-Y ratio:

$$\left(\frac{I}{Y}\right)^* = \frac{\nu}{u_n}(g_Z + \delta) \tag{15}$$

The denominator of the supermultiplier in Eq. (13) is thus equal to the difference between the propensity to save and the investment share in output.[6] The positive impact of g_Z on the level of the supermultiplier thus results from the accelerator effect: any increase in g_Z generates pressures on production capacity, to which firms respond in the long run by revising their growth expectations through Eq. (6).

According to Eqs. (14) and (15), an increase in g_Z results in both a decrease in the autonomous consumption-to-output ratio and an increase in the investment-to-output ratio. Skott (2019) views this outcome as counterintuitive, undermining the supermultiplier models, which should lead to their disqualification. In contrast, Girardi and Pariboni (2016, 2020) and Pérez-Montiel and Manera (2022) provide empirical results that are consistent with this outcome. We revisit the issue of the composition of aggregate demand after splitting Z into two subcomponents.

4. Autonomous components in the long run: an attempt at clarification

According to authors who are skeptical of supermultiplier models, this approach is undermined by the fact that the growth rate of the autonomous non-capacity-generating demand component (g_Z) can hardly remain exogenous in the long run. For example, Skott (2019) argues that "to be autonomous, a component of aggregate demand (…) must be exogenous (i.e., independent of other variables in the model, including aggregate income and employment)" (p. 234), while Nikiforos (2018) states that "in the long run, it is unlikely that 'autonomous expenditure' is really autonomous" (p. 661). Proponents of supermultiplier models respond by asserting that "in the long run there is no truly exogenous variable" (Lavoie, 2016, p. 194) and that "'autonomous' need not mean 'exogenous' or 'constant'" (Fazzari et al., 2020, p. 20). This potential endogeneity of autonomous components leads Fiebiger (2018, 2020) and Fiebiger and Lavoie (2019) to suggest replacing the term 'autonomous' with 'semi-autonomous', arguing that "critics […] have wondered whether any component of effective demand would ever be fully autonomous in the real world; hence, the prefix of semi as found in Kalecki (1968) seems preferable. Obviously, no one believes that the growth rate of the semi-autonomous expenditures would be a constant value in the real world, even on average" (Fiebiger and Lavoie, 2019, p. 251). However, these answers do not convince skeptical authors, as illustrated by Nikiforos' (2018) comment that "the prefix 'semi' makes a world of difference" (p. 671, fn. 15).

In an attempt to clarify, we first propose to remind that the short-run induced components are endogenous by construction. Moreover, the nature of this endogeneity is clearly specified: the level of these components is determined as a proportion of the current level of income, which explains why they are engaged in the circular flow of income at work in the multiplier process. Therefore, their growth rate is given by the growth rate of income.

Now, the logical requirement to distinguish between induced and autonomous components can lead to the corollary that "the autonomous components must be defined as all demand components that are not induced" (Allain, 2021, p. 619). Such a condition applies of course to exogenous components whose levels and dynamics are entirely determined outside the model, as is often assumed for exports, for example.

However, following Skott and Nikiforos criticisms, it is questionable whether any demand component can be viewed as totally exogenous in the long run. Most likely none. For example, according to models of export-led cumulative causation, any increase in the domestic income can generate a technical progress (Verdoorn's law), then an improvement of competitiveness, which in turn causes an increase in exports.[7] Exports are therefore partly induced by domestic income, which should not make us forget that they also remain partly autonomous (depending on foreign income in particular). Of course, a proper theoretical specification should split the exports into two parts: the induced part should be included in the induced consumption (C) taking into account the consequences on the propensity to save; only the autonomous part should remain in Z. If this partition is not done (for instance, because of the difficulty to specify the exports as the sum of two separate functions), then exports can be considered as a semi-autonomous component. As a consequence, although the value of their growth rate can be influenced by the growth rate of domestic income (g_Y), it is not given by g_Y as would be the case if exports were induced component. The same considerations can apply to some other autonomous components. For example, although households' residential investment (Fiebiger, 2018; Fiebiger and Lavoie, 2019) partly depends on income, it also depends on demographics and financial innovations. Similarly, basic needs (Allain, 2019, 2021) depend on demographics and labor productivity. These two demand components can be considered semi-autonomous as long as demographics, financial innovations and labor productivity remain at least partly exogenous with respect to the level and dynamics of income.

To summarize, for a component to be autonomous or semi-autonomous in the long run, its growth rate must not be given by that of income (g_Y). Nevertheless, since both income and all the demand components must grow at the same rate, this raises the tricky question of causality. Causality runs unambiguously from g_Z to g_Y if g_Z is exogenously determined by foreign income, demographics, financial innovations, etc. However, what can we say if g_Z is subject to a discretionary decision? We answer this question with the example of public spending (G), assuming that the government discretionary set the growth rate g_G. A source of confusion may arise from the fact that this discretionary choice has no reason to be independent of the current and expected values of g_Y. However, the fact that 'it is not independent of' must not be confused with the statement that 'it is given by' the current or expected values of g_Y. Clearly, if a Keynesian government considers that the expected value of g_Y is too low, it can set g_G at a higher level. If the system behaves as in the model of Section 3, then the causality runs unambiguously from g_Y to g_G, and government expenditure must be considered as autonomous or semi-autonomous although the decision

[6] Notably, the neo-Kaleckian version of the supermultiplier is isomorphic to the Sraffian version according to which, by definition, the investment-to-output ratio is given by the propensity to invest, which is equal to h^* in the long-run equilibrium. Indeed, in Sraffian as in neo-Kaleckian models, the traverse relies on a Harrodian adjustment of investment. In the Sraffian version, this adjustment involves the propensity to invest according to the following behavior: $\dot{h} = h\varphi(u - u_n)$, where φ stands for the adjustment parameter. The long-run propensity to save is the value h^* for which $u = u_n$. As a result, the supermultiplier corresponds to $Y_t = Z_t/(s - h^*)$. See Freitas and Serrano (2015) among others.

[7] See Blecker (2013) and Blecker and Setterfield (2019) for recent presentations of these models.

on g_Y 'is not independent of' the expected value of g_Y.

Nikiforos (2018, p. 661) addresses a caveat here as he reminds us that funding for demand components should not be overlooked. Since the autonomous components remain unspecified in our general framework, this issue cannot be examined as well as it should be. Nevertheless, the competition among demand components to access income through saving and other leakage (taxes, imports, etc.) provides simple, albeit crude, indications of financing conditions. Indeed, $g_Y < g_Z$ and/or a rise in Z/Y can indicate increasing difficulties in financing Z, which would require a decrease in the value of g_Z. The question of whether the demand component should then be considered induced in the long run remains open because the value of the decrease in g_Z can be partially exogenous: for instance, adjusting g_Z to g_Y is a discretionary decision if nothing prevents g_Z from being set to a value lower than g_Y. Conversely, $g_Y > g_Z$ and/or a fall in Z/Y can indicate increasing financing opportunities, which would allow an increase in the value of g_Z but not make it required; Z should therefore still be considered as semi-autonomous.

Now, it is worth noting that, if Z has only one component (say government expenditure) and if the model behaves as in Section 3, an increase in g_G is not expected to generate funding difficulties because it leads to a new equilibrium such that $g_Y^* = g_G$ while $(G/Y)^*$ has decreased.[8] In this case, financial conditions do not require that fiscal austerity replaces the fiscal stimulus. Government expenditure remains autonomous. This question will be reexamined with the introduction of two semi-autonomous components in the following sections.

5. Considering several non-capacity-generating semi-autonomous demand components

The questions addressed by Skott and Nikiforos become more challenging if several non-capacity-generating semi-autonomous components are taken into account. Indeed, it is obvious that different autonomous components cannot grow indefinitely at different paces, as those with lower rates of growth will eventually be eliminated. Formally, assuming that Z is split into two subcomponents, Z_1 and Z_2, whose growth rates are g_1 and g_2, the only consequences for the short-run equilibrium and dynamics are

$$u = \frac{\nu(\gamma + \delta - \gamma_u u_n + z_1 + z_2)}{s - \nu\gamma_u} \tag{16}$$

$$\dot{u} = \frac{\nu(\dot{\gamma} + \dot{z}_1 + \dot{z}_2)}{s - \nu\gamma_u} \tag{17}$$

where $z_{i=1,2} = Z_i/K$ and $\dot{z}_{i=1,2} = z(g_i - g_K)$. On the other hand, the consequences for long-run equilibrium are

$$g_Y^* = \gamma^* = g_K^* = g_1^* = g_2^* = g_Z \tag{18}$$

$$z_1^* = \sigma z^*, \; z_2^* = (1-\sigma)z^* \tag{19}$$

$$\frac{Z_1}{Y} = \sigma\left(\frac{Z}{Y}\right)^*, \; \frac{Z_2}{Y} = (1-\sigma)\left(\frac{Z}{Y}\right)^* \tag{20}$$

Note that the value of z^* in Eq. (19) is still given by Eq. (10) and that, at this stage, the steady state is not fully identified since the parameter $\sigma \in [0,1]$ remains unknown. This problem is overcome in the simulations, where its value is exogenously set in the initial period.

According to Eq. (18), a critical question is whether or not Z_1 and Z_2 can remain autonomous while having the same growth rate. This issue has been circumvented in three ways in the existing theoretical literature. First, the vast majority of models neglect the problem by focusing on a single component (government expenditure, for example). Second, some models consider the autonomous component as an aggregate that combines several subcomponents, but the respective dynamics of these subcomponents are not analyzed (Fazzari et al., 2020). Finally, only two papers explicitly assume the presence of two autonomous components. However, they bypass the risks of divergent dynamics by assuming that the growth rates of these two components are equal to each other in the long run. Thus, Freitas and Christianes (2020) assume that "the rates of growth of the two autonomous demand components [government expenditures and capitalist consumption] are related in a way that, although they can be different, there is a tendency for them to be equal, on average, over time" (p. 319).

Morlin (2022), who focuses on exports and public expenditure, adopts the same strategy, although he assumes that the two growth rates (g_X and g_G) can differ from each other in the short or medium run. For instance, the author shows that the foreign debt is likely to increase when both the G-to-Z ratio (noted σ) and the propensity to import are relatively high. To prevent the foreign debt from exceeding a ceiling, the government must implement a temporary decrease in g_G, which reduces the value of σ and thus allows convergence toward the ceiling. However, g_G then gradually returns to the long-run exogenous growth rate given by g_X. In an alternative scenario, Morlin (2022) assumes that the government implements industrial policies to increase the income elasticities of exports and generate a permanent increase in g_X. This increase in g_X loosens the constraints on the government (both the foreign debt and the public debt decline), which enables an adjustment of g_G on g_X. In fact, the government's choice is ultimately limited since it must adjust g_G to the new value of g_X for the model to reach a long-run equilibrium. In summary, Morlin's (2022) model does not resolve the issue addressed above because both g_Y and g_G are finally given by g_X. Exports should thus be considered the single autonomous component in the long run, while public expenditure behaves as an induced component.

One lesson that can be drawn from the above is that the decision to give the label 'autonomous' to a demand component should not depend on the nature of an expenditure as in the list proposed in Serrano (1995a) or on an argument such as "the component is financed by credit". This label can only result from the formal properties of the component in the theoretical model. This conclusion also raises questions about empirical analyses that are based on the aggregation of several components (exports, government expenditures, residential investment, credit-financed consumption, etc.) for which it is not sure that they are genuinely autonomous.[9] However, their conclusion that "Y is more Granger-caused by Z than the reverse" can be interpreted as supporting the supermultiplier approaches, especially since Granger causality going from Z to Y ought to be biased downward by the presence of such non-genuine components in Z (and conversely, Granger causality going from Y to Z ought to be biased upward).

Moreover, empirical analysis promotes a more pragmatic approach than theoretical analysis because one cannot expect the causality going from Y to Z to be zero. Thus, according to Girardi and Pariboni (2016), "the fact that Z is autonomous means that it is not determined by output through a necessary functional relation. Even so, Z does not fall from the sky: it is socially and historically determined; among the various social and economic factors that influence autonomous spending, economic growth certainly plays a major role" (p. 535). This influence of income can be due to the fact that empirical data do not distinguish between the induced and the autonomous parts of demand components (as suggested above when evoking the export-led cumulative causation hypothesis). It can also be due to the fact that discretionary decisions are related to the economic situation (as we have already seen for government expenditure).

Nevertheless, keeping Eq. (18) in mind, the problem remains to make the assumption that the components are autonomous consistent with the

[8] According to Hein and Woodgate (2021), g_Z should not fall short of the interest rate to avoid public debt unsustainability if interest payment is introduced in the model.

[9] See Girardi and Pariboni (2016, 2020), Pérez-Montiel and Manera (2022).

requirement that their growth rates converge to each other in the long run. We cannot provide a detailed demonstration, especially in this 'general' model where we have chosen not to specify Z_1 and Z_2 to remain focused on conceptual issues. The following simulations have therefore been implemented to illustrate different cases in order to clarify the long-run dynamics and thus allow a better understanding of supermultiplier approaches.

6. Numerical calibration

The simulations were performed using the parameter calibration that is provided in Table 1. We did not conduct a sensitivity analysis because our goals are limited to illustrating and comparing different conceptual cases rather than demonstrating the validity of the theoretical model or making policy recommendations.

The normal rate of capacity utilization (u_n) is that of the United States between 1980 and 2007 (see Nikiforos, 2016). Fazzari et al. (2020) computed an actual value of the capital-to-output ratio of 1.2 between 1980 and 2016. Combined with $u_n = 0.79$, this leads to a capital to full-capacity output ratio of $\nu = 0.948$. Both δ and s are also set in accordance with Fazzari et al. (2020). It should be emphasized that, in the model, s does not only refer to the saving behavior of households (in which case $s = 0.5$ would be highly overestimated) since it also must include taxes and imports.

It is quite easy to see that $\gamma_u = 1/u_n$ would instantly close the gap between u and u_n in Eq. (4).[10] The short-run speed of adjustment is therefore measured by the product $\gamma_u u_n \leq 1$. Setting $\gamma_u = 0.25$ means that $\gamma_u u_n = 0.1975$ so that firms adjust their investment to close only one-fifth of the gap between the current capital stock and the level that would be needed to produce at the normal rate of capacity utilization.

The system is initially assumed to be in its steady state, so that all components (C, I, Z_1 and Z_2) grow at the same rate of 3%. Moreover, the initial value of σ is set to 0.5, which means that the two autonomous or semi-autonomous components, Z_1 and Z_2, have the same weight. Under these assumptions, Eq. (10) allows calculating $z^* = 0.303$ and the shares of C, I, Z_1 and Z_2, which are 50.0%, 13.7%, 18.2% and 18.2%, respectively.

The value of the speed of adjustment of entrepreneurs' expectations (ψ) is set according to the stability condition that $\psi < z^*$ (Eq. (12)). On this basis, the choice of ψ is subject to a tradeoff: if the value of ψ is high, the duration of the oscillation period (between two dates at which $u = u_n$) is short, but the amplitude of the oscillations is great (i.e., the difference between u_n and the extreme values taken by u). Moreover, since $\psi < z^*$ corresponds to a local condition, it no longer holds when u is far from u_n. Therefore, if ψ is too high (while remaining below z^*), then the oscillations are explosive, and the model becomes unstable. Here, $\psi = 0.4 \times z^* = 0.121$ means that entrepreneurs try to fill approximately one-eighth of the gap between γ and g_K in each period.

Such a value would probably not convince Skott (2017), who considers that "it would seem reasonable to assume that expected growth has closed half the gap (…) within something like 2 years", which leads him to suggest that ψ should be equal to 0.35. From his point of view, a value of ψ that is too low is a serious limitation to the relevance of supermultiplier models. At least two responses can be made to Skott's criticisms. On the theoretical side, first, Skott (2017) offers no argument to support his 'reasonable assumption' of a rapid adjustment, while a relatively low value of ψ can be justified on economic grounds. Indeed, according to Freitas and Serrano (2015, p. 270), a "drastic adjustment [in γ through ψ] is highly unrealistic, both because of the durability of fixed capital (which means that producers want normal utilization only on average over the life of equipment and not at every moment) and because producers know that demand fluctuates a good deal and therefore do not interpret every fluctuation in demand as indicative of a lasting change in the trend of demand". Second, on the side of simulations, it must be pointed out that those proposed by Skott et al. (2021) have been run with the unfunded assumption that $u_n = 0.5$. Due to this odd assumption, the upper bound on the stability condition in Eq. (12) is $\psi = 0.050$. Following the tradeoff explained above, $\psi = 0.030$ is destabilizing, which leads the authors to set $\psi = 0.015$. This value is eight times lower than that in our simulations. In other words, Skott et al. (2021) consider that firms close only 1.5% of the gap between g_K and γ in each period. It is therefore not surprising that their simulations show such a low convergence, with several decades (or even centuries) being necessary for the system to stabilize.

Moreover, we fully agree with Lavoie's response to Skott's (2017) criticism: our theoretical model should be considered a "*prototype*, as it leaves aside several key determinants of economic activity and abstract from the world complexity" (Lavoie, 2017, p. 195). Therefore, simulations can hardly be used to refute a theoretical model, mainly because there are no epistemic rules regulating such use. Simulation exercises primarily provide an illustration of the types of dynamics that can be generated by a given set of assumptions, which is the objective of the following exercises.

Table 1
Parameters and initial values of variables for simulations.

$s =$	$u_n =$	$\nu =$	$\delta =$	$\gamma_u =$	$\psi =$	$\sigma =$
0.5	0.79	0.948	0.084	0.25	0.121	0.5

7. Model simulations: exogenous long-run equilibrium rate of growth

In the first four simulations, the system is assumed to leave the steady growth path when g_1 is exogenously increased to 4% while g_2 remains equal to 3%. In Scenario A, the gap between the two rates is maintained throughout the entire period analysis. Fig. 1 shows the resulting dynamics of the rate of capacity utilization (u), while Fig. 2 shows the dynamics of the growth rates of aggregate income (g_Y), capital accumulation (g_K), and aggregate autonomous component (g_Z) with

$$g_Z = \sigma g_1 + (1 - \sigma) g_2 \tag{21}$$

Because of the sudden surge in g_1, g_Z jumps to 3.5% before beginning a regular increase as the weighting parameter σ increases. Indeed, σ reaches 0.52 after 10 periods and 0.62 after 50 periods: the growth of Z is both driven by Z_1 and burdened by Z_2. As predicted by the theoretical model, all growth rates are attracted by g_Z (which is the so-called 'supermultiplier effect'), and the rate of capacity utilization returns to its normal level. According to the principle of effective demand, the former reaction to the increase in g_1 is an increase in output (both u and g_Y increase). The increase in the rate of accumulation is first the result of the increase in u. Only then do firms reconsider their expected growth rate (γ) through Eq. (6). The rate of capacity utilization increases during 7 periods to a peak slightly above 0.81 and then returns to its normal value through dampened oscillations. Because of the accelerator effect, the oscillations of g_K are larger than those of g_Y. However, the gap with g_Z never exceeds 0.25 percentage points for g_Y (at the 7th period) and 0.46 percentage points for g_K (at the 12th period). Dampened oscillations then allow convergence toward g_Z.

In this scenario, as in the other ones, the time required for the system to reach its steady state may seem too long. However, it can be pointed out that firms' adjustment of the γ parameter (Eq. (6)) is based on naïve adaptive expectations that are well known to stretch the stabilization time of any model. According to Fiebiger (2021), Eq. (6) could be

[10] Combining $u = Y/Y_{fc}$ with $\nu = K/Y_{fc}$ results in $u = \nu Y/K$. Noting K_n the level of capital stock that would enable the production at the normal utilization rate u_n, we get $u = \nu Y/K$ and $u_n = \nu Y/K_n$. Therefore, the rate of capital accumulation required to reach K_n is given by $\frac{K_n}{K} - 1 = \frac{1}{u_n}(u - u_n)$ which means that γ_u must be equal to $1/u_n$ in Eq. (4) to allow such an instantaneous adjustment.

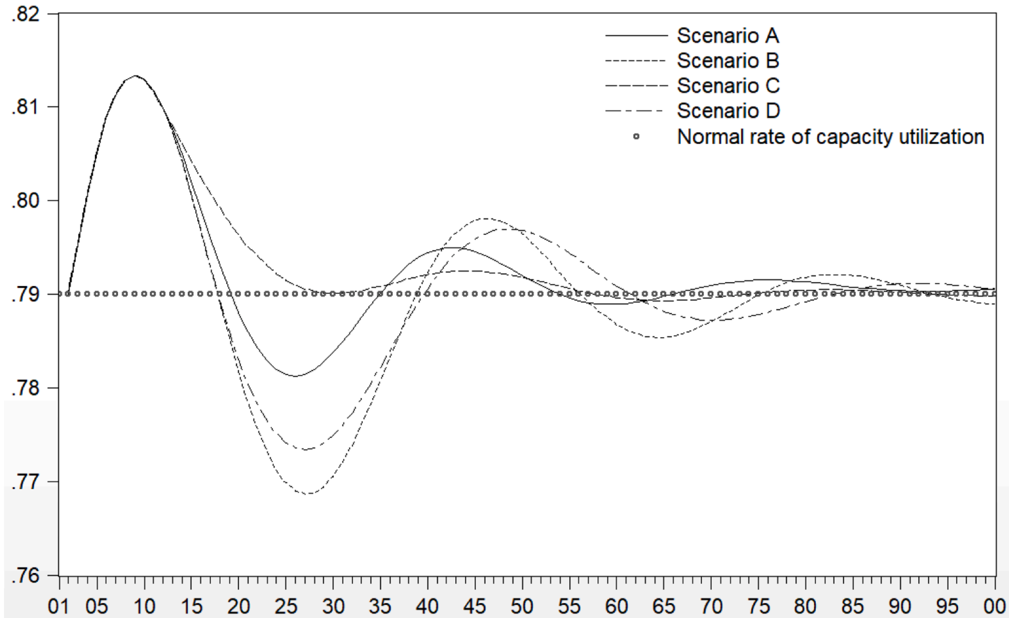

Fig. 1. Dynamics of the rate of capacity utilization (all scenarios).

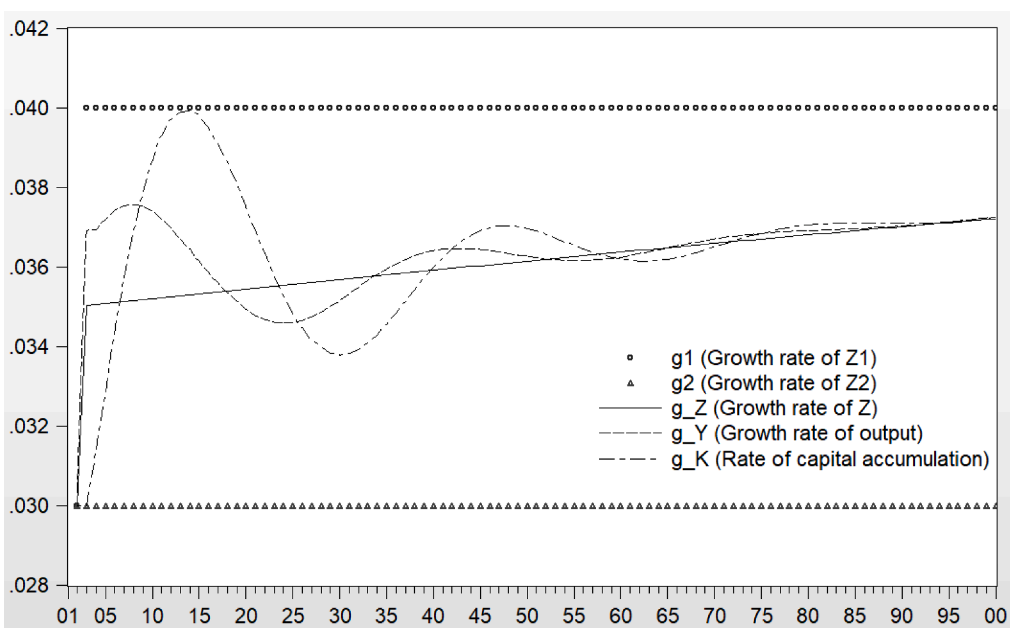

Fig. 2. Dynamics of the growth rates (scenario A).

referred to as an 'ultra-Harrodian' adjustment since "firms seem to believe that the secular sales growth trend lacks any persistence. Such firms form expectations with myopia about the possibility of counter-cyclical macro stabilization policies" (p. 13). The author then shows that the convergence is significantly improved under the alternative assumptions of partial or fully anchored adjustments.[11] Moreover, the figures may give the impression that stabilization is effective only when $u = u_n$ and $g_K = g_Y = g_Z$. In fact, these conditions are never met in reality since the economy is always subject to many different shocks. We therefore could admit the existence of a range within which the model is considered stabilized despite $u \neq u_n$ and $g_K \neq g_Y \neq g_Z$.

Nevertheless, in scenario A, while the share of induced consumption in output remains constant over time (by construction, $C/Y = 50.0\%$), the increase in g_Z is accompanied by an increase in I/Y (from 13.7% to 14.5% after 50 periods), which is offset by an equal decrease in Z/Y (from 36.3% to 35.5%). However, the latter change dissimulates more important divergences since Z_1/Y increases from 18.2% to 22.0% while Z_2/Y decreases from 18.2% to 13.6% (over 50 periods). These outcomes (which will also be observed with some nuances in the other scenarios) help to respond to Skott's (2019) criticism that supermultiplier models predict a change in the output shares of autonomous components in the wrong direction: according to Eq. (14), any increase in g_Z would result in a decrease in Z/Y, which would be completely at odds with the

[11] Note that the simulations implemented by Fiebiger (2021) confirm that the 'ultra-Harrodian' adjustment we use leads to convergences that are incomparably faster than those performed by Skott et al. (2021) under the assumption that $\psi = 0.015$.

unsurprising facts that “the military buildup during the Reagan years raised the share of defense in GDP just as the housing boom in the 2000s raised the share of construction in GDP” (Skott, 2019, p. 241). However, Skott interprets the real economy while overlooking the presence of several autonomous components in Z. Indeed, the simulations clearly show that an increase in g_1 leads to an increase in Z_1/Y, which is more than offset by the decrease in Z_2/Y (hence the decrease in Z/Y). Thus, the output shares do not move in what Skott (20019) considers the wrong direction.

In the very simplified model above, it is obviously not possible to determine how long the divergent dynamics of g_1 and g_2 can last. What is certain is that if they continue indefinitely, Z_2 will converge to zero, and only Z_1 will remain.[12] This is not what is usually observed, indicating that some mechanisms lead to a readjustment of g_1, g_2 or both. As suggested in Section 4, such readjustments could be due either to discretionary decisions, or to the fact that Z_1 and Z_2 still contain an induced part (beside their autonomous part), or to changing financial conditions. Although these three types of readjustment cannot be distinguished in our very general framework, the following simulations will make it possible to envisage different cases. For this, starting from scenario A, we will consider that after a while there is either a decrease in g_1 (scenario B), an increase in g_2 (scenario C), or a combination of both changes (scenario D). Of course, in a fully specified model, these adjustments would depend on the nature of each component. In our general model where Z_1 and Z_2 are not specified, we get around the difficulty with the following function:

$$\dot{g}_{it} = \rho_i \phi_i \left(g^e_{Yt} - g_{it-1} \right) \qquad (with\, i = 1, 2) \tag{22}$$

where $\rho_{i=1,2}$ is a dummy variable. If $\rho_i = 0$, there is no adjustment, and g_i must be considered exogenous. In contrast, $\rho_i = 1$ means that g_i converges gradually (at an adjustment speed corresponding to $\phi_{i=1,2} < 1$) to the expected growth rate of income g^e_Y. The advantage of Eq. (22) is to capture in a simple behavioral function the many endogenous and exogenous adjustments that are required for the system to reach its long-run equilibrium. However, its major drawback is to present in a too simple way a behavior whose complexity has been discussed in Section 4. For example, as already pointed out, if the agent in charge of g_i is faced with financial constraints, he may either adjust g_i to g_Y or decide on a target lower than g_Y.[13] Moreover, the shifts in ρ_i can be endogenously required by such financial constraints, but there are many situations in which the value of ρ_i remains exogenous (for example, if $g_i < g_Y$ the agent may either shift ρ_1 to 1 or keep $\rho_1 = 0$). For the sake of simplicity, we set $\phi_{i=1,2} = \psi = 0.121$.

In scenario B, g_1 initially jumps from 3% to 4%, but the dummy variable ρ_1 is assumed to turn to 1 after 10 periods. A possible explanation of this switch relates to the growing difficulty of funding Z_1 that results from the slow convergence of g_Y to g_1 and the increase in Z_1/Y: for example, since public spending increases faster than taxes, public debt rises, prompting the government to reduce g_1; or, as the growth of credit-financed residential investment exceeds that of household income, the risk exposure of banks increases, leading them to decide on a cut in g_1. From this shift, g_1 converges back gradually to g_Y. However, the value of this target decreases over time as g_2 remains unchanged. Eventually, the system stabilizes when all the growth rates return to 3% (see Fig. 3). This is the scenario that causes the largest amplitude of oscillations in the rate of capacity utilization (Fig. 1).[14] This is due to the so-called ‘ultra-Harrodian’ adjustment whereby firms need time before becoming aware that the long-run growth rate has remained unchanged. Moreover, the income shares of C, I and Z return to their initial levels (50.0%, 13.7% and 36.3%, respectively). However, because $g_1 > g_Y > g_2$ on average, Z_1’s share in income increased from 18.2% to 20.1%, while Z_2’s share decreased from 18.2% to 16.2%. Note that the question of whether Z_1 should be considered induced or semi-autonomous will be addressed at the end of the section.

Scenario C is somewhat symmetrical to scenario B: g_1 remains equal to 4%, while the dummy variable of the other subcomponent (ρ_2) is assumed to turn to 1 after 10 periods. A possible explanation of this switch relates to the funding opportunities of Z_2 resulting from the fact that $g_2 < g_Y$ and Z_2/Y decreases over time: for example, the decrease in public debt and deficit prompts an increase in the growth rate of government expenditure; or, the improvement in the financial situation of households encourages banks to speed up the supply of credit.[15] As a result, all the growth rates are gradually attracted by g_1 (see Fig. 4). This is the scenario that causes the smallest amplitude of the oscillations in the rate of capacity utilization (Fig. 1). This is because firms are now revising their growth expectations in the right direction from the beginning. Moreover, since the steady growth path is changed ($g_Z = 4\%$), the income share of I increases (from 13.7% to 14.9%), while that of Z decreases (from 36.3% to 35.1%). However, as previously, because $g_1 > g_Y > g_2$ on average, Z_1’s share in income increased (from 18.2% to 19.4%), while Z_2’s share decreased (from 18.2% to 15.8%).

At first glance, it would be tempting to consider that, in scenario B, Z_1 is induced in the long run while Z_2 remains autonomous. Similarly, it would be tempting to consider that the reverse occurs in scenario C. However, scenarios B and C are two different outcomes of the same situation. Clearly, the financing of Z_1 becomes increasingly difficult as $g_1 > g_Y$ and Z_1/Y increases over time. A downward adjustment in g_1 is therefore required (scenario B), except if the growing funding opportunities for Z_2 lead to an (unrequired) increase in g_2, which will fuel economic growth and ease the access to income for Z_1 (scenario C). This means that the question of whether a demand component should be considered induced in the long run also depends on the dynamics of the other components, an argument for continuing to consider Z_1 and Z_2 as semi-autonomous.

8. Model simulations: path-dependent long-run equilibrium rate of growth

At this stage, a mix of scenarios B and C seems particularly relevant to combine the decrease in the growth rate of components facing potential financial difficulties with the increase in the growth rate of components facing financial opportunities. This is the purpose of scenario D, in which ρ_1 and ρ_2 are set to 1 after 10 and 15 periods, respectively: all the growth rates now converge toward a steady state of 3.2% (Fig. 5), which is path dependent because the causality between $g_{i=1,2}$ and g_Y runs both ways. This steady state value results from a large set of parameters, including the initial values of g_1 and g_2, the speeds of convergence (ϕ_1 and ϕ_2) and the weights of Z_1 and Z_2. Of course, any change in the value of these parameters would result in a different value of the long-run growth path.

[12] At this ultimate stage, we would have $Z_1/Y = Z/Y = 35.1\%$, which means that, despite the decrease in Z/Y, Z_1’s share in income will have increased from 18.2% to 35.1%.

[13] With respect to discretionary decisions, the rationale behind Eq. (22) is that agents can control the growth rate of semi-autonomous components (g_i) while they cannot control the share of these components in income (Z_i/Y) because Y is endogenous. Of course, the specification of the nature of the demand components (government expenditure, exports, residential investment, etc.) would lead to a more appropriate reformulation of Eq. (22).

[14] Note that the length of the oscillations is not significantly different from one scenario to the other.

[15] Scenario C could also illustrate the increase in the growth rate of exports resulting from the increase in the domestic income growth rate through a cumulative causation mechanism. However, the elasticity of exports with respect to domestic income is likely to be significantly less than unity. As a result, despite the increase in g_2, the gap with g_1 cannot be filled in the absence of other adjustment mechanisms.

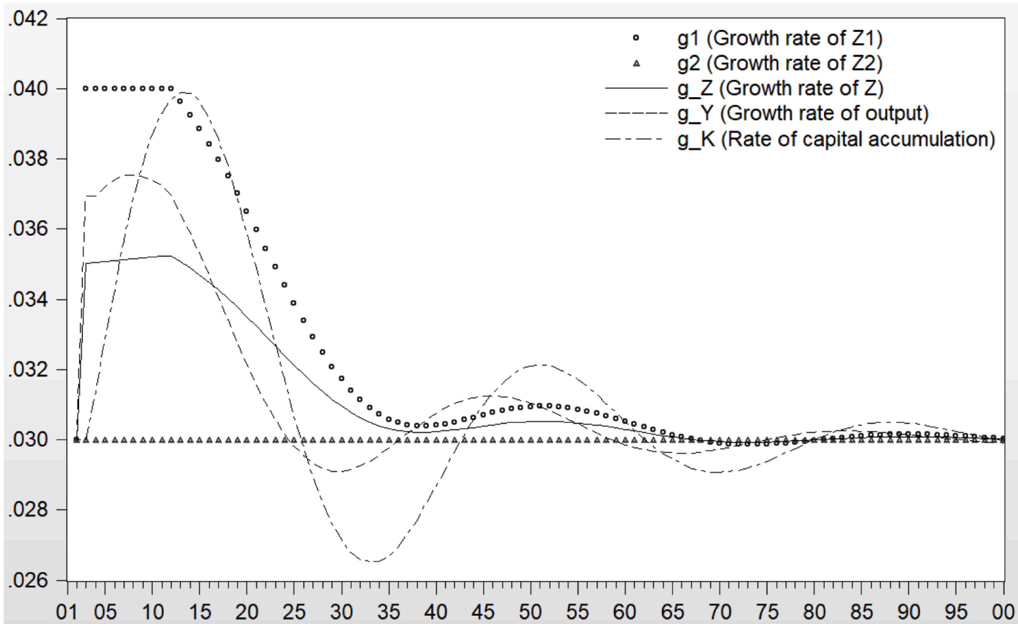

Fig. 3. Dynamics of the growth rates (scenario B).

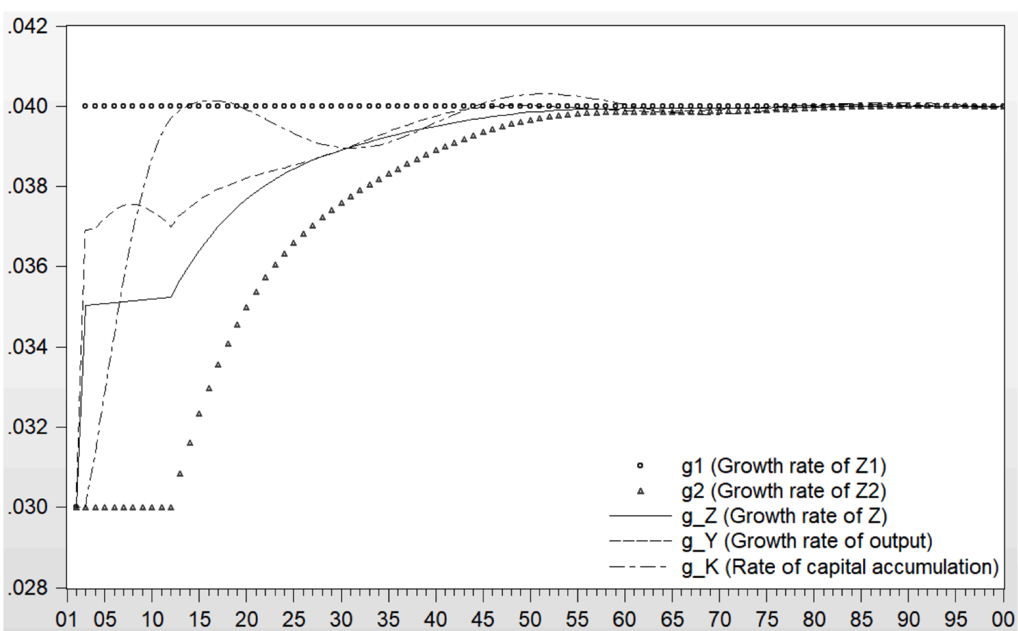

Fig. 4. Dynamics of the growth rates (scenario C).

The path dependency is likely to explain why the duration of oscillations is slightly longer than in the previous scenarios, since the equilibrium value of g_Y now results from the mutual convergence of g_1 and g_2 . Nevertheless, the dynamics of the rate of capacity utilization are quite similar to those of the other scenarios (Fig. 1). In addition, the income share of I increases (from 13.7% to 14.0%), while that of Z decreases (from 36.3% to 36.0%). Once again, as $g_1 > g_Y > g_2$ on average, Z_1's share in income increased (from 18.2% to 19.8%), while Z_2's share decreased (from 18.2% to 16.3%).[16]

The possibility of a path-dependent long-run growth rate is clearly a new finding in the literature on supermultiplier models.[17] Indeed, in scenario D, the equilibrium growth rate of 3.2% is due to the past

[16] What some economists consider as a weakness may finally appear as a desirable property of supermultiplier models: in scenario D, the rather slow convergence speeds (ϕ_1 and ϕ_2) is a necessary condition to avoid the system instability.

[17] It is worth noting that path dependency here is limited to growth rates while the normal rate of capacity utilization remains exogenous. Thus, our model differs significantly from those proposed by Lavoie (1996), Dutt (1995), and Hein *et al.* (2012) among others, according to which it is the normal rate of capacity utilization that is primarily subject to path dependency.

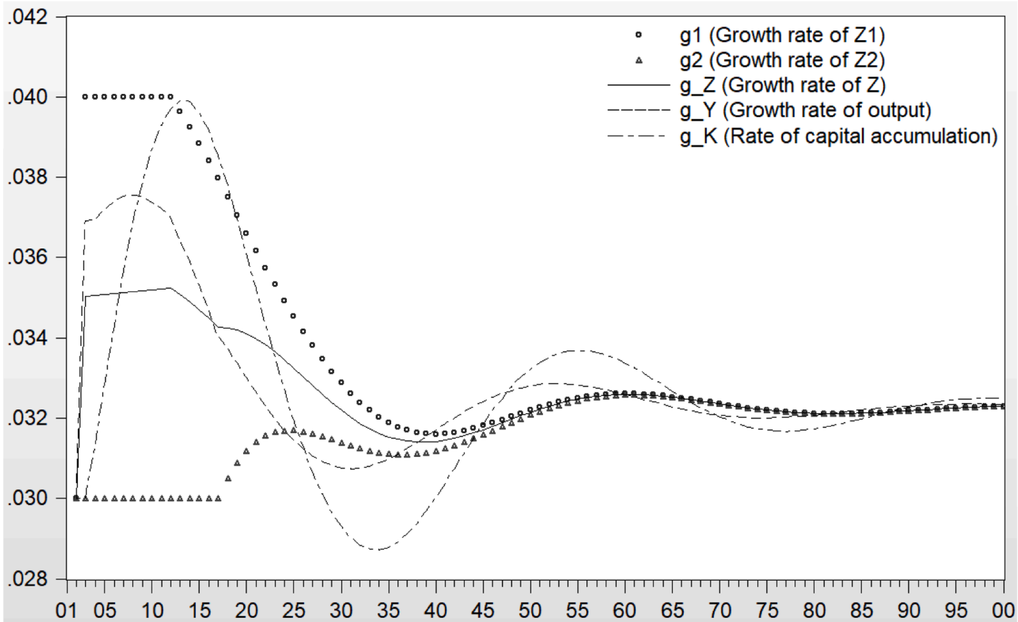

Fig. 5. Dynamics of the growth rates (scenario D).

dynamics of g_1 and g_2; it results from a divergence that occurred in previous periods. To illustrate this case, assume that the government wants to implement a policy to increase the long-run growth rate of the economy. To do so, it must take into account that there are several demand components of aggregate demand, some of which adjust to income slowly or only partially (e.g., exports). The government must therefore implement a significant increase in the growth rate of its expenditure over a sufficiently long period to be able to drive the sticky demand components. However, after a while the government reduces its effort because both its financial situation deteriorates and the rise in the growth rate of the other endogenous components is too slow (due to a low domestic income elasticity of exports, for example). The growth rate of government expenditure converges back to the growth rate of income, but the latter is now greater than the initial situation. To draw a parallel with physical sciences, we could say that it requires a large torque to move a system subject to inertia.[18]

Another illustration might be an increase in credit-financed consumption (for residential investment, for instance) followed by a slowdown as banks become aware of the deteriorating financial situation of borrowers. In the meantime, however, the growth rate of other semi-autonomous components could have increased (exports, as above, or also public spending if the government does not decide to deleverage). Here, again, the steady growth path of the economy is permanently higher than the initial one.[19]

Scenario D proposes an alternative outcome of the same situation that was simulated in scenarios B and C. Although g_1 and g_2 are now endogenous, it remains appropriate to consider the two components as semi-autonomous because there is no theoretical requirement to maintain $\rho_1 = \rho_2 = 1$. Indeed, one of the dummy variables can be switched to 0 at any moment, which would lead us back to scenario B or C. Another lesson that can be drawn here is that the usual assumption in literature that the growth rate of the autonomous component is exogenous could be relaxed or confined to a simplifying assumption. Eventually, a rather unexpected outcome of scenario D is the return of the strict version of the paradox of thrift: because of path dependency, any increase in the propensity to save leads to a decrease in the growth rate of income, which in turn causes a decrease in both g_1 and g_2.

To conclude on simulations, the presence of path dependency calls for a return to the firms' animal spirits (the γ parameter in Eq. (4)). Indeed, Eq. (9) states that, in the long run, g_K^* and γ^* must be equal to g_Z, a condition that is achieved through the supermultiplier effect and the Harrodian behavior of firms Eq. (6)). As long as g_Z is assumed to be exogenous, γ^* is endogenous, and investment must be considered an induced capacity-generating component in the long run. However, since g_Z becomes endogenous in scenario D, this raises the question of whether firms' animal spirits can contribute to such a path dependency. Scenario E has thus been implemented under the two following assumptions. First, capital accumulation is given by Eqs. (4) and ((6), except over a few periods in which it is governed by exogenous animal spirits ($\overline{\gamma} =$ 4%):

$$I = \overline{\gamma}K \tag{23}$$

Second, the non-capacity-generating components adjust according to Eq. (22) assuming that $\rho_i = 1$. The results are shown in Figs. 6 and 7. Investment first fuels aggregate demand, which is reflected in the

[18] The example also provides a response to Skott et al. (2021) who oddly claim that, according to supermultiplier models, if a government targets a growth rate of output (say, g_Y^T), it should set the growth rate of its own expenditure to g_Y^T and wait decades and even centuries for the system to reach that target. Of course, such scenario would undermine the supermultiplier approaches. However, the above-alleged behavior is their pure invention. The irony is that what they propose as an alternative (a functional finance behavior of government) is fully consistent with a supermultiplier model: the government must first massively boost its expenditure before releasing its effort; in the long run, however, the growth rate of income is given by the growth rate of public spending.

[19] Note that, in cases where the agents' financial situation has an impact on the growth rate of a semi-autonomous component, this impact is likely to be asymmetric: if a financial deficit requires a decrease in g_i, an excess of resources does not necessarily lead to an increase in g_i. As a result, the system dynamics are more likely affected by the components that impede than by those that stimulate economic growth.

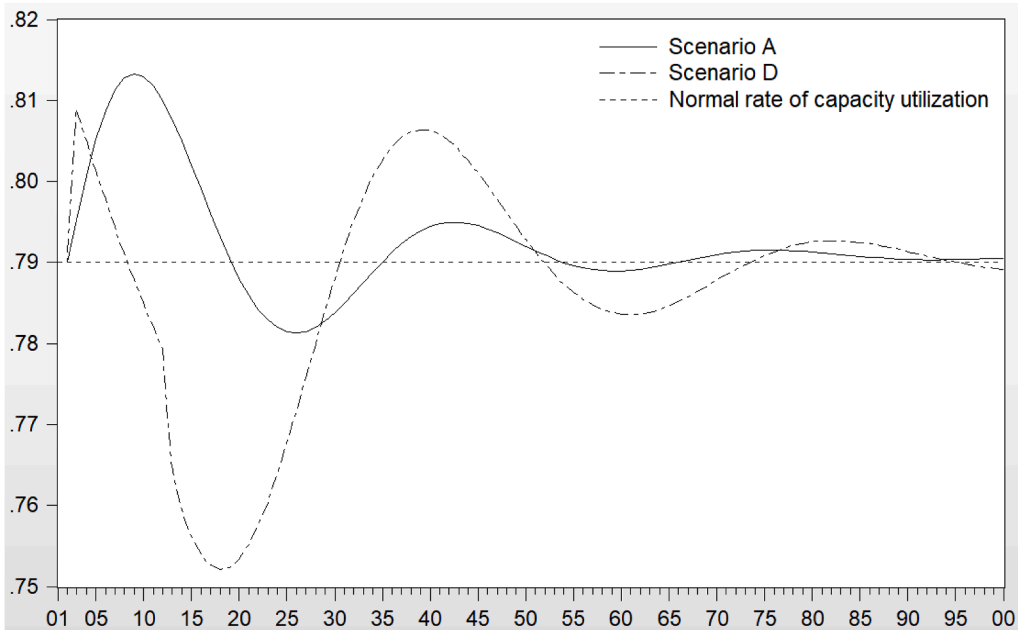

Fig. 6. Dynamics of the rate of capacity utilization (scenario E).

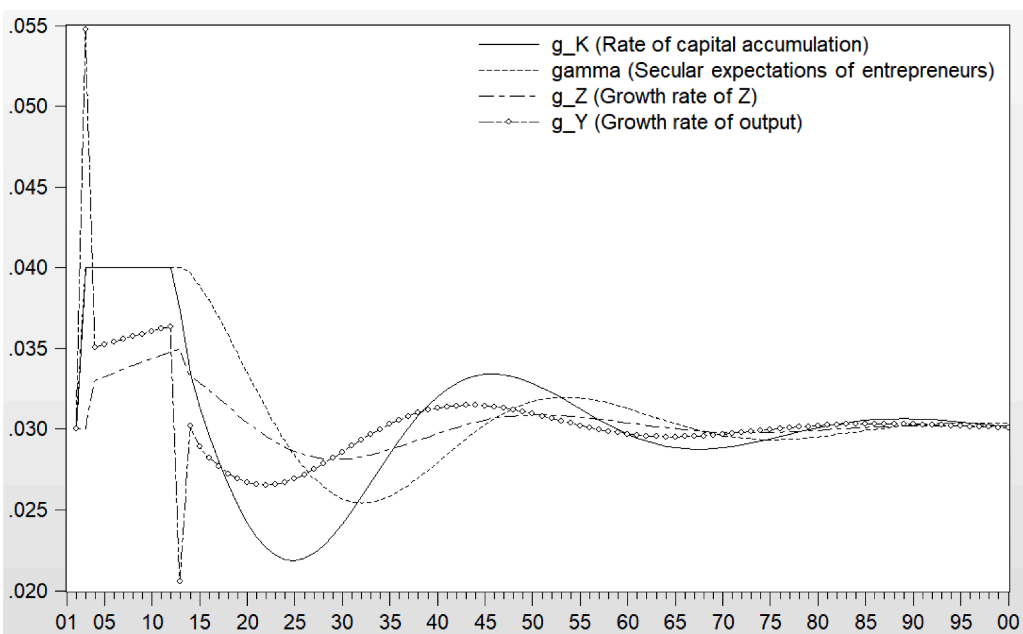

Fig. 7. Dynamics of the growth rates (scenario E).

increase in u and g_Y. The two semi-autonomous components also respond positively to this impulse. However, the rate of capacity utilization soon falls because capital accumulation outpaces economic growth ($g_K > g_Y > g_Z$). The idle capacity is thus substantial after 10 periods, when firms revert to the behavior described in Eqs. (4) and (6). As a result, capital accumulation continues at a low rate, driving down economic growth and the growth of the semi-autonomous components. Eventually, the system stabilizes at a rate that is not significantly different from 3%. Accordingly, such simulations support the conclusion that investment should indeed be considered an induced component in the long run.

9. Concluding remarks

Because existing supermultiplier models suggest several candidates as non-capacity-generating autonomous demand components (government expenditures, credit-financed consumption, private residential investments, primary needs consumption, etc.), the question arises as to how two or more autonomous components can coexist in the long run. To answer this question, we first attempt to clarify the distinction between induced and autonomous components. On the one hand, the level of induced components is specified as a proportion of current income, which explains why they are engaged in the circular flow of income at work in the multiplier process. Therefore, their growth rate is given by the growth rate of income. On the other hand, the autonomous components are the injections that initiate the multiplier process. However, as suggested by Fiebiger (2018, 2020) and Fiebiger and Lavoie (2019) through the term 'semi-autonomous', these autonomous components can be endogenous in the long run in at least three ways. First, some components can combine an induced and an autonomous part, which may be kept together because of theoretical or empirical difficulties in distinguishing one from the other. As a result, the growth rate of these components can be influenced by the growth rate of income without being given by the latter (as opposed to induced components). This can be the case for exports under the export-led cumulative causation hypothesis. This also can be the case for households' residential investment (Fiebiger, 2018; Fiebiger and Lavoie, 2019) and basic needs (Allain, 2019, 2021).

Second, the growth rate of some components, such as government expenditure or residential investment, can be the result of discretionary choices. There is however no inconsistency to consider that these growth rates are related to, but not given by, the growth rate of income.

Finally, the financing conditions can affect the growth rate of the demand components. Funding difficulties (when the growth rate of the component is higher than the growth rate of income) can require a downward adjustment. Conversely, financing opportunities (when the growth rate of the component is lower than the growth rate of income) may prompt an upward adjustment. However, as we argued in the text, the components should still be considered semi-autonomous.

In this paper, we also carried out different simulation exercises based on a general theoretical framework including two semi-autonomous components. In the first three scenarios, only one component remain exogenous in the long run: either the one with the lower growth rate ultimately vanishes (scenario A), or endogenous adjustments lead to the convergence of the growth rate of one component toward that of the other (scenario B and C). However, the adjusting components can still be considered as semi-autonomous rather than induced, in part because the economic system does not dictate which remains exogenous and which has to adjust: the latter can be the one with the highest growth rate as well as the other.

Moreover, scenario D shows that the steady growth path can be path dependent, which occurs through the mutual convergence between the income growth rate and the growth rates of each semi-autonomous components. In this case, any variation in the growth rate of a semi-autonomous component over a few periods can have a lasting, albeit attenuated, impact on the steady growth path. Again, the demand components involved should be considered semi-autonomous, as the mutual convergence is not required by the economic system, which can adapt through the alternative scenarios presented above. Note that this outcome provides a rebuttal of Blecker and Setterfield's (2019, p. 366) claim that supermultiplier models bring heterodox growth theories back to exogenous rather endogenous dynamics. This rebuttal is also evidenced by the model proposed in Brochier and Silva (2019), in which the growth rate of the autonomous component is endogenous, and by Morlin's (2022) attempt to introduce export-led cumulative causation in a supermultiplier framework. Eventually, a conclusion that can be drawn from scenario D is that the usual assumption in the literature that the growth rate of autonomous component is exogenous should be relaxed or confined to a simplifying assumption.

Regarding the use of supermultiplier models for analytical purposes, arguments have been made against Skott's (2019) criticism that the convergence time to reach the steady state is far too long in these models. In particular, the simulations implemented by Skott et al. (2021) greatly overestimate the length of convergence. Although several periods are required for the system to stabilize, this property does not undermine the analytical relevance of supermultiplier models. Indeed, the simulations confirm that, even over short periods, both the growth rate of income and the rate of capital accumulation are attracted by the weighted average growth rate of semi-autonomous components (g_Z). Moreover, the simulations also suggest that economic dynamics result from the opposite impact of the semi-autonomous components that stimulate growth and those that slow it down. In addition, while there may be exogenous reasons for trend reversals, one must also consider the path dependency that can result from endogenous and mutual adjustments over some periods. As a consequence, supermultiplier models can be used "to help explaining specific periods, episodes or modes of accumulation (and the seeds of forthcoming crises within them) as, for instance, the consumer debt-led growth of the 'Great Moderation' era that preceded the 'Great Recession' or the German export-led mercantilist model and the European financial unbalances that it created" (Pariboni, 2016, p. 222).[20] Such a proposition could be interpreted as an abandonment of long-run considerations in favor of short-run ones. However, the undeniable contribution of the theoretical model is to show that this framework can be relevant in the short run because the system is not subject to long-run knife-edge instability. To put it another way, long-run considerations are compatible with the idea that "life is a traverse" (Halevi and Kriesler, 1992, p. 229).[21]

The model presented in this article may be frustrating because of its high degree of generality. Since the semi-autonomous components (Z_i) are not specified, the endogenous mechanisms leading to the mutual convergence of their growth rates have been suggested rather than closely examined. Therefore, the next step of the research will be to develop a supermultiplier model with two specified semi-autonomous components (exports and government expenditure, for example) and to analyze its long-run properties, particularly its ability to generate a path-dependent equilibrium.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authorship statement

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All persons who meet authorship criteria are listed as authors, and all authors certify that they have participated sufficiently in the work to take public responsibility for the content, including participation in the concept, design, analysis, writing, or revision of the manuscript. Furthermore, each author certifies that this material or similar material has not been and will not be submitted to or published in any other publication before its appearance in the *Hong Kong Journal of Occupational Therapy*.

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Category 1

Conception and design of study: O. Allain, ________, ________, ________; acquisition of data: no data, ________, ________, ________; analysis and/or interpretation of data: no data, ________, ________, ________.

Category 2

Drafting the manuscript: O. Allain, ________, ________, ________; revising the manuscript critically for important intellectual content: O. Allain, ________, ________, ________.

Category 3

[20] See also Freitas and Dweck (2013) and Morlin et al. (2021).

[21] See also Thompson (2022) who proposes an alternative approach in which, although stable equilibria need not exist, non-capacity-generating components can determine the long-run rate of expansion of the economy.

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Data Availability

No data was used for the research described in the article.

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